



# Distribution cost optimization: Comparison of NWC, MODI, and Stepping Stone methods in transportation problems

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## ABSTRACT

Solving transportation problems is essential in minimizing distribution costs in logistics and supply chains. Three classical methods North West Corner (NWC), Modified Distribution Method (MODI), and Stepping Stone are frequently used, but few studies offer a comprehensive comparison. This study fills this gap by evaluating their performance using simulated data representing real-world distribution scenarios. This study applies a structured comparative framework to analyze NWC (a cost-agnostic initial allocation technique), MODI (a dual-variable-based optimization approach), and Stepping Stone (a closed-loop path evaluation method). Each method was tested on a simulated cost matrix using Python. Evaluation metrics included total distribution cost, number of iterations, and computation time. The NWC method yielded a feasible but suboptimal solution with a cost of 540 units. Optimization using MODI reduced the cost to 425, while Stepping Stone further minimized it to 410 after three iterations. MODI showed greater computational efficiency, while Stepping Stone offered visual traceability of cost reductions. This study contributes methodologically by combining heuristic and iterative optimization techniques in one analytical framework. Practically, it provides decision-makers with insights into selecting appropriate solution methods based on trade-offs between simplicity, efficiency, and cost minimization.

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## 1. INTRODUCTION

Suboptimal distribution can lead to increased logistics costs, delayed deliveries, and decreased customer satisfaction levels [1][2]. One systematic approach used to achieve this efficiency is through modeling transportation problems [3]. Transportation issues have become an important foundation in logistics planning and distribution decision making [4]. The transportation problem is an optimization technique that is modeled precisely using linear programming by calculating an initial basic feasible solution (IBFS), which is then optimized [5]. IBFS is an important step to achieve the minimum total cost (optimal solution) of a transportation problem [6]. Transportation Problem (TP) is concerned with transporting a number of products from source to destination at minimum total cost, provided that demand and supply constraints can be met [7]. Various classical methods have been developed to solve transportation problems, such as the simplest initial method, namely the North West Corner (NWC) [8] [9] [10]. To improve the initial solution, the Modified Distribution Method (MODI) is used

[11] [12] [13]. If negative values are still found in the cell evaluation, then iterations of improvement are carried out until the optimal condition is reached. Meanwhile, the Stepping Stone method uses a closed-path approach to assess the impact of possible new allocations to empty cells on the total distribution cost [4][14]. Modified Distribution Method (MODI) and Stepping Stone Method are the most acceptable methods in finding minimum total cost solutions for transportation problems [15]. Classical methods such as North West Corner (NWC), Modified Distribution Method (MODI), and Stepping Stone are often used separately in previous studies without a comprehensive comparative study of the aspects of cost efficiency, number of iterations, and computational burden.

Although the NWC, MODI, and Stepping Stone methods have long been used in solving transportation problems, most existing research only compares the two methods in pairs or focuses on a single implementation in a specific context. Research by [16][17][18][19][20] only comparing NWC with other approximation methods without involving further optimization analysis, comparing NWC with other initial solution analysis methods to determine the effectiveness of the NWC method compared to other initial solution methods [21][5]. Analysis of transportation problems using the Vogel Approximation and MODI methods [22] Vogel's Approximation Method (VAM) with MODI [23] [11].

The lack of studies that integrate the accuracy, iteration efficiency, and computational complexity of the three methods simultaneously creates a gap in the scientific literature that this study can fill. Therefore, a comprehensive comparative analysis between the NWC, MODI, and Stepping Stone methods is urgently needed to provide theoretical and practical contributions to efficient, data-driven distribution decision-making. This study fills this gap by conducting a comprehensive evaluation of all three methods simultaneously using computational simulation and experimental approaches.

This study aims to analyze and compare the effectiveness and efficiency of three transportation problem solving methods, namely NWC, MODI, and Stepping Stone, in the context of distribution cost optimization. This research makes a significant scientific contribution to the field of operations research by presenting a comprehensive evaluation of three classical methods for solving transportation problems, namely NWC, MODI, and Stepping Stone. These findings are expected to form the basis for more rational and data-driven decision-making, especially in selecting the most appropriate and practical optimization method according to the characteristics of the distribution problem faced. In addition to enriching the academic literature on transportation optimization, this study also contributes to bridging the gap between theoretical approaches and application needs in the field.

## 2. RESEARCH METHOD

This study uses a comparative quantitative approach that aims to analyze and compare the effectiveness and efficiency of three transportation problem-solving methods, namely North West Corner (NWC), Modified Distribution Method (MODI), and Stepping Stone. To ensure objective and replicable results, this study was designed through a data simulation process that represents real distribution conditions, the use of valid computational tools, and measurable quantitative evaluation criteria. All stages of the research are systematically outlined in the following subsections, including simulation design, data types and sources, computational tools, evaluation metrics, validation schemes, and procedural flow modeling in the form of diagrams.

### a) Simulation Design

This study uses a comparative quantitative approach based on simulation. The transportation matrix is simulated involving 4 sources and 4 destinations, representing the distribution scenarios presented in Table 2. This data is compiled to reflect real-world conditions, including supply, demand, and distribution costs between sources and destinations. This allows for testing the performance of the method in the context of distribution within transportation problems. The transportation matrix is presented in the format shown in Table 1.

Table 1. general representation of the transportation problem in matrix form

Source	D <sub>1</sub>	D <sub>2</sub>	...	D <sub>n</sub>
S <sub>1</sub>	C <sub>11</sub>	C <sub>12</sub>	...	C <sub>1n</sub>
S <sub>2</sub>	C <sub>21</sub>	C <sub>22</sub>	...	C <sub>2n</sub>
S <sub>3</sub>	C <sub>31</sub>	C <sub>32</sub>	...	C <sub>3n</sub>
⋮	⋮	⋮	⋮	⋮
S <sub>m</sub>	C <sub>m1</sub>	C <sub>m2</sub>	...	C <sub>mn</sub>
Demand	b <sub>1</sub>	b <sub>2</sub>	...	b <sub>3</sub>

Source: [5]

**b) Data Type and Source**

The type of data used is quantitative data, developed through Python scripts to replicate distribution scenarios. The data source comes from a controlled experimental design to evaluate the efficiency and effectiveness of three methods, NWC, MODI, and Stepping Stone, on transportation problems with the following equation:

$$\begin{aligned}
 & \sum_{i=1}^m \sum_j^n X_{ij} C_{ij} \\
 & \sum_{i=1}^m X_{ij} = a_i, i = 1, 2, \dots, m \\
 & \sum_{i=1}^m X_{ij} = b_j, j = 1, 2, \dots, n \\
 & \sum_{i=1}^m > 0 \text{ for all } i = 1, 2, \dots, m \text{ and } j = 1, 2, \dots, n
 \end{aligned}
 \tag{1}$$

- 1) Data collection: Collecting distribution cost, supply, and demand data as a basis for calculations.
- 2) Initial Basic Feasible Solution (IBFS): Determine a feasible initial solution to meet demand and supply.
- 3) Distribution Matrix: Form a distribution matrix based on the initial solution.
- 4) Initial Solution with North West Corner (NWC): Determine the initial solution using the northwest corner method.
- 5) Calculate the Difference Between Rows and Columns: Calculate the difference value as a basis for optimization.
- 6) Solution Optimization (MODI): Using the Modified Distribution (MODI) method to evaluate the optimality of the initial solution.
- 7) Optimal Decision: Check whether the solution is optimal; if so, proceed to model analysis.
- 8) Solution Optimization using Stepping Stone: If it is not optimal, use the Stepping Stone method to calculate cost changes and improve the solution.
- 9) Model Analysis Test: Testing the optimization model results using Matlab software.

**c) Evaluation Metrics**

The evaluation was based on two main aspects, namely effectiveness and efficiency. Effectiveness was measured by the total distribution cost generated by each method. Efficiency was assessed based on the number of iterations required and the computation time (execution time) to achieve the optimal solution. In addition, the size of the instance (number of source and destination points) was also recorded to measure the relative complexity of the cases analyzed.

**d) Validation Scheme**

To ensure the accuracy of the results, validation was performed by comparing the manual solutions from the MODI and Stepping Stone methods with the solutions from the linprog() function in the SciPy library in Python. This approach ensures that the manual optimization results obtained have reached the optimal condition in accordance with the mathematical solution based on linear programming. Python was chosen due to its flexibility in implementing numerical algorithms, as well as its support for optimization libraries such as NumPy, SciPy, and PuLP, to ensure that the results obtained from the manual method align with those from the linear programming-based solver. Python was selected for its ability to automate calculations, efficiency in iterative processes, and ease of data visualization.

**e) Flowchart of Procedure**

The analytical steps in this study are described in the form of a research flowchart as shown in Figure 1.

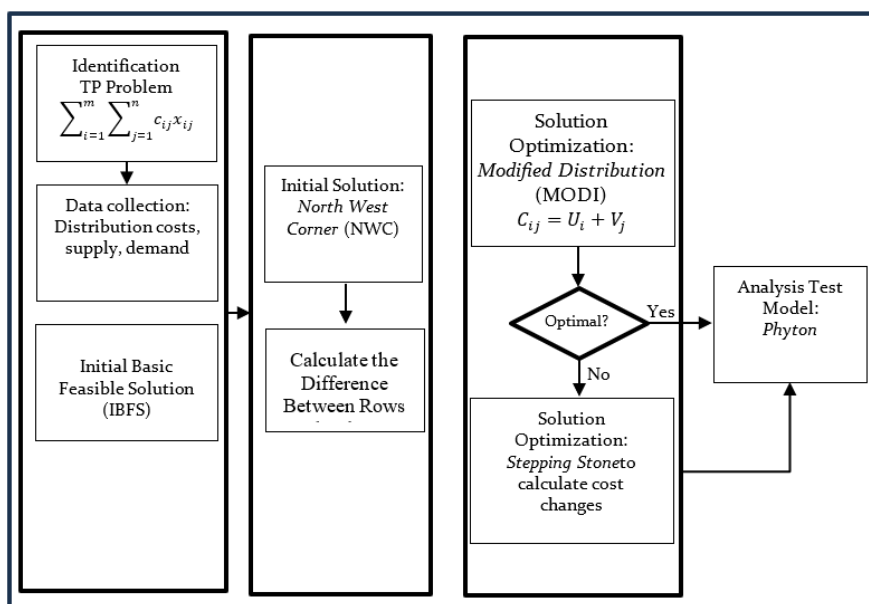


Figure 1. Flowchart of transportation problem optimization using NWC, MODI, and Stepping Stone

**3. RESULTS AND DISCUSSIONS**

**3.1. Numerical Example**

Penelitian ini dimulai dengan menyusun *distribution matrix* seperti pada Tabel 2 yang merepresentasikan biaya pengangkutan antara masing-masing sumber (S<sub>1</sub>-S<sub>4</sub>) dan tujuan (D<sub>1</sub>-D<sub>4</sub>), lengkap dengan data suplai dan permintaan. Tahapan ini bertujuan untuk memformulasikan masalah transportasi ke dalam bentuk matematis yang dapat diselesaikan dengan metode optimasi.

Table 2. Distribution matrix of the transportation problem (numerical example)

Source	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>	Supply
S <sub>1</sub>	7	5	9	11	30
S <sub>2</sub>	4	3	8	6	25

$S_3$	3	8	10	5	20
$S_4$	2	6	7	3	15
Demand	30	30	20	10	90

**Stage 1: NWCR Process**

The North West Corner method is used to generate an initial basic feasible solution (IBFS). The process starts from the north-west cell of the distribution table and performs maximum allocation based on supply and demand values. The goal is to ensure all demand and supply are met without considering costs.

- a) Starting from the cell ( $S_1, D_1$ ):  
 Supply, Demand  $S_1 = 30, D_1 = 10$   
 Location: 30  
 →  $S_1$  finished, move to the next row  $S_2$
- b) Furthermore :( $S_2, D_2$ )  
 $S_2 = 25, D_2 = 30$   
 Allocation: 25  
 →  $S_2$  finished, move to the next row  $S_3$
- c) ( $S_3, D_2$ ):  
 $S_3 = 20, D_2$  sisa = 5  
 Allocation: 5  
 →  $D_2$  finished, move to the next row  $D_3$
- d) ( $S_3, D_3$ ):  
 $S_3 = 15, D_3 = 20$   
 Allocation: 15  
 →  $S_3$  finished, move to the next row  $S_4$
- e) ( $S_4, D_3$ ):  
 $S_4 = 15, D_3$  sisa = 5  
 Allocation: 5  
 →  $D_3$  finished, move to the next row  $D_4$
- f) ( $S_4, D_4$ )  
 $S_4 = 10, D_4 = 10$   
 Allocation: 10  
 →  $S_4$  finished, finished

Table 3. Initial allocation using north west corner method (NWCR)

	$D_1$	$D_2$	$D_3$	$D_4$	Supply
$S_1$	30				0
$S_2$		25			0
$S_3$		5	15		0
$S_4$			5	10	0
Demand	0	0	0	0	

Total transportation cost (cost)  
 $= (30 \times 7) + (25 \times 3) + (5 \times 8) + (15 \times 10) + (5 \times 7) + (10 \times 3) = 210 + 75 + 40 + 150 + 35 + 30 = 540$   
 Final Answer:

Allocation matrix: as above.  
 Total initial transportation cost (NWCR): 540

The initial allocation results are shown in Table 3, with an initial total cost of 540. While this solution is mathematically feasible, it is not necessarily the most cost-efficient solution.

**Stage 2: MODI Method (Modified Distribution Method)**

Once the initial solution is obtained, the next step is to evaluate its optimality using the Modified Distribution Method (MODI). The aim is to find out if the initial solution can be improved to reduce the total cost. This process involves calculating the row and column potential values ( $u_i$  and  $v_j$ ), as well as calculating  $\Delta_{ij}$  for each empty cell. If  $\Delta_{ij} < 0$  is found, then the solution is not optimal.

Do the following steps:

- 1) Calculate the value  $u_i$  (row) and (column) based on:  $v_j$   
 $C_{ij} = u_i + v_j$  for filled cells only
- 2) Count  $\Delta_{ij} = C_{ij} - (u_i + v_j)$  for all empty cells
- 3) If all  $\Delta_{ij} \geq 0$ , the solution is optimal. If there is  $\Delta_{ij} < 0$ , then it is not optimal.

Continue stepping stone

Table 4. Evaluation of initial solution using MODI (modified distribution method)

	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>
S <sub>1</sub>	30			
S <sub>2</sub>		25		
S <sub>3</sub>		5	15	
S <sub>4</sub>			5	10

Form equation based on Filled cells

$$(S_1, D_1): 7 = u_1 + v_1$$

$$(S_2, D_2): 3 = u_2 + v_2$$

$$(S_3, D_2): 8 = u_3 + v_2$$

$$(S_3, D_3): 10 = u_3 + v_3$$

$$(S_4, D_3): 7 = u_4 + v_3$$

$$(S_4, D_4): 3 = u_4 + v_4$$

Step: Solve the System of Equations

Example:  $u_1 = 0$  (free to set as a reference)

$$u_1 = 0 \rightarrow v_1 = 7 \text{ (dari } 7 = u_1 + v_1)$$

$$(S_2, D_2): u_2 + v_2 = 3$$

$$(S_3, D_2): u_3 + v_2 = 8 \rightarrow \text{dari (2) dan (3): } u_3 = u_2 + 5$$

$$(S_2, D_3): u_3 + v_3 = 10 \rightarrow v_3 = 10 - u_3 = 10 - (u_2 + 5) = 5 - u_2$$

$$(S_4, D_3): u_4 + v_3 = 7 \rightarrow u_4 = 7 - v_3 = 7 - (5 - u_2) = 2 + u_2$$

$$(S_4, D_4): u_4 + v_4 = 3 \rightarrow v_4 = 3 - u_4 = 3 - (2 + u_2) = 1 - u_2$$

We can choose the value  $u_2 = 1$

So

$$u_1 = 0$$

$$v_1 = 7$$

$$u_2 = 1$$

$$v_2 = 2$$

$$u_3 = 6$$

$$v_3 = 1$$

$$u_4 = 3$$

$$v_4 = 0$$

Count  $\Delta_{ij}$  for empty cells

$$\Delta_{ij} = C_{ij} - (u_i + v_j)$$

Table 5. Evaluation of opportunity costs ( $\Delta_{ij}$ ) in MODI method

Empty Cell	$C_{ij}$	$u_i + v_j$	$\Delta_{ij}$
(S <sub>1</sub> , D <sub>2</sub> )	5	0 + 2	3
(S <sub>1</sub> , D <sub>3</sub> )	9	0 - 1	10
(S <sub>1</sub> , D <sub>4</sub> )	11	0 + 0	11
(S <sub>2</sub> , D <sub>1</sub> )	4	1 + 7	-4 !
(S <sub>2</sub> , D <sub>3</sub> )	8	1-Jan	8
(S <sub>2</sub> , D <sub>4</sub> )	6	1 + 0	5
(S <sub>3</sub> , D <sub>1</sub> )	3	6 + 7	-10 !
(S <sub>3</sub> , D <sub>4</sub> )	5	6 + 0	-1 !
(S <sub>4</sub> , D <sub>1</sub> )	2	3 + 7	-8 !
(S <sub>4</sub> , D <sub>2</sub> )	6	3 + 2	1

Because there is  $\Delta < 0 \rightarrow$  meaning it is not optimal  
 Select the most negative cell:  $\Delta (S_3, D_1) = -10$

**Stage 3: Stepping Stone Method**

This stage is done to improve the solution based on MODI evaluation. By utilizing the Stepping Stone Method, a closed cycle of empty cells with negative  $\Delta$  is created, following the pattern of + and - signs. The main objective of this stage is to optimize the allocation to obtain the minimum total cost. This process is shown in the calculation of total cost change (-5), and the improved result is shown in Table 6 as the new allocation matrix.

Initiation Cell: (S<sub>3</sub>, D<sub>1</sub>)

Steps: Create a square cycle (loop) from and to these empty cells through the filled cells.

Loop (alternating + and - signs):

(S<sub>3</sub>, D<sub>1</sub>)[sel kosong]  $\rightarrow (+)$

(S<sub>1</sub>, D<sub>1</sub>)  $\rightarrow (-)$

(S<sub>1</sub>, D<sub>3</sub>)[sel kosong]  $\rightarrow (+)$

(S<sub>3</sub>, D<sub>3</sub>)  $\rightarrow (-)$

Loop: (S<sub>3</sub>, D<sub>1</sub>) $\rightarrow$  (S<sub>1</sub>, D<sub>1</sub>) $\rightarrow$  (S<sub>1</sub>, D<sub>3</sub>) $\rightarrow$  (S<sub>3</sub>, D<sub>3</sub>) $\rightarrow$

Calculate the total loop cost

$$\text{Total Change in Cost} = +3(S_3, D_1) - 7(S_1, D_1) + 9(S_1, D_3) - 10(S_3, D_3)$$

$$= 3 - 7 + 9 - 10 = -5$$

Cost reduction: -5, meaning the solution can be optimized

Update all allocations in the loop according to the mark

(S<sub>3</sub>, D<sub>1</sub>): +15

(S<sub>1</sub>, D<sub>1</sub>): -15  $\rightarrow$  15

(S<sub>1</sub>, D<sub>3</sub>): +15

(S<sub>3</sub>, D<sub>3</sub>): -15  $\rightarrow$  0

New Matrix

Table 6. Allocation matrix after second iteration using stepping stone method

	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>
S <sub>1</sub>	15		15	
S <sub>2</sub>		25		
S <sub>3</sub>	15	5	0	
S <sub>4</sub>			5	10

New Fees

$$(15 \times 7) + (15 \times 9) + (25 \times 3) + (15 \times 3) + (5 \times 7) + (10 \times 3) = 105 + 135 + 75 + 45 + 35 + 30 = 425$$

After initial optimization using the MODI method and one iteration of Stepping Stone, there are still some empty cells in the final distribution matrix, namely  $(S_3, D_1)$ ,  $(S_2, D_2)$ ,  $(S_2, D_3)$ ,  $(S_2, D_4)$ ,  $(S_3, D_2)$ ,  $(S_3, D_4)$ ,  $(S_4, D_2)$ , dan  $(S_4, D_3)$ .

From the calculation of  $\Delta_{ij}$ , it is known that cells  $(S_4, D_1)$  and  $(S_4, D_2)$  have negative  $\Delta$  values, indicating further cost saving opportunities. Therefore, the iteration process is continued to improve the solution based on the empty cells that have negative  $\Delta$ .

If in the optimality evaluation using the MODI method there are still empty cells with negative  $\Delta_{ij}$  values, it indicates that the existing distribution solution is not optimal. A negative  $\Delta_{ij}$  value indicates that if an allocation is made to the cell, the total distribution cost can be reduced. Therefore, it is necessary to perform further iterations using the Stepping Stone method. This step starts by selecting the empty cell that has the most negative  $\Delta_{ij}$  value as the starting point, then forming a closed path (loop) with other filled cells, following the pattern of alternating positive and negative signs. Once the path is formed, an allocation adjustment is made based on the minimum value of the negatively marked cell in the loop. The allocation at each cell in the loop is then updated by adding the value at the positive sign position and subtracting the negative sign position. This process is continued iteratively until there are no more negative  $\Delta_{ij}$  values in empty cells. This condition indicates that there are no more redistribution paths that can produce cost efficiency, so the solution obtained can be declared as the optimal solution and the iteration can be stopped. Thus, the presence of negative  $\Delta_{ij}$  values is an important indicator in making decisions to continue or stop the distribution optimization process.

The further iteration procedure is done in the same way as the first iteration, namely:

- Identifying the empty cell with the most negative  $\Delta_{ij}$  value.
- Constructing a closed path (loop) involving the empty cell and other filled cells in an alternating pattern of + and - signs.
- Determine the minimum allocation at the negative cell in the loop as the change boundary.
- Update the distribution allocation in the direction of the loop.
- Recalculate the total distribution cost after the change.

This iteration is performed repeatedly as long as there is still  $\Delta_{ij} < 0$ , as this indicates that the solution is not optimal. When all  $\Delta_{ij}$  values for empty cells are zero or positive, then the iteration can be stopped, because there are no more distribution improvements that can reduce costs.

In the 3rd iteration, the optimization path of cell  $(S_4, D_1)$  is obtained again, and after the update, all  $\Delta_{ij}$  values for empty cells are  $\geq 0$ . Therefore, the 3rd iteration is the last iteration and the resulting solution is declared as the optimal solution.

Table 7. Final optimal distribution matrix after third iteration (stepping stone method)

	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>	Supply
S <sub>1</sub>	15	5	10	-	30
S <sub>2</sub>	-	25	-	-	25
S <sub>3</sub>	10	-	10	-	20
S <sub>4</sub>	5	-	-	10	15
Demand	30	30	20	10	

The total cost is calculated by summing up the results of multiplying the allocation by the cost of each unit according to the initial cost table, the total cost calculation is as follows:

$$\begin{aligned}
 &= (15 \times 7) + (5 \times 5) + (10 \times 9) + (25 \times 3) + (10 \times 3) + (10 \times 10) + (5 \times 2) + (10 \times 3) \\
 &= 105 + 25 + 90 + 75 + 30 + 100 + 10 + 30 \\
 &= 410
 \end{aligned}$$

The following table summarizes the decrease in distribution costs for each iteration:

Table 8. Decrease in distribution cost in each iteration

Iteration	Method Used	Total Cost (Units)	Reduction from Previous Iteration	Reduction from Initial Cost
Initial Solution	North West Corner (NWC)	540	-	-
Iteration 1	Stepping Stone ( $\Delta S_3, D_1$ )	425	115	115
Iteration 2	Stepping Stone ( $\Delta S_1, D_2$ )	420	5	120
Iteration 3 (Final)	Stepping Stone ( $\Delta S_4, D_1$ )	410	10	130

Table 8 summarizes the process of reducing distribution costs during the optimization iterations conducted in this study. The initial cost of 540 unit costs was obtained from the initial solution using the North West Corner (NWC) method, which only considers allocation based on position without considering cost efficiency. After the first iteration using the Stepping Stone method, by identifying an empty cell ( $S_3, D_1$ ) as the optimization point, the total cost was significantly reduced to 425, resulting in a saving of 115 cost units or about 21.3% of the initial cost.

The process continued to the second iteration, where a new redistribution path through cell ( $S_1, D_2$ ) was found and applied. As a result, the total cost again dropped to 420, providing an additional 5 unit cost reduction from the previous iteration. The third and final iteration was conducted with a fix point at an empty cell ( $S_4, D_1$ ), which reduced the cost to 410 cost units. Thus, there is an additional saving of 10 cost units from the second iteration.

Overall, the optimization process through three iterations of Stepping Stone resulted in a total decrease in distribution costs by 130 cost units from the initial solution, or equivalent to more than 24% savings. The third iteration is the last iteration because all  $\Delta_{ij}$  values in empty cells have been zero or positive, indicating that the solution obtained has reached the optimal condition and can no longer be improved further.

To verify the accuracy of the manual calculations, the same transportation data was tested using Python's `linprog()` function. This computational approach helps determine whether the manual solution is optimal or requires further improvement.

```

import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
from scipy.optimize import linprog

# ----- STEP 1: Data Setup -----
costs = [
    7, 5, 9, 11, # S1
    4, 3, 8, 6, # S2
    3, 8, 10, 5, # S3
    2, 6, 7, 3 # S4
]

supply = [30, 25, 20, 15]
demand = [30, 30, 20, 10]

# Objective function (cost vector)
c = np.array(costs)

# ----- STEP 2: Constraint Building -----
A_eq = []
b_eq = []

# Supply constraints (rows)
for i in range(4):
    row = [0] * 16
    for j in range(4):
        row[i*4 + j] = 1
    A_eq.append(row)
    b_eq.append(supply[i])

```



```

plt.figure(figsize=(6, 4))
sns.heatmap(allocation, annot=True, fmt=".0f", cmap="YlGnBu", cbar=False,
            xticklabels=["D1", "D2", "D3", "D4"],
            yticklabels=["S1", "S2", "S3", "S4"])
plt.title(f"Optimal Allocation Matrix\nMinimum Total Cost: {int(total_cost)}", fontsize=12)
plt.tight_layout()
plt.savefig("optimal_allocation_heatmap.png", dpi=300)
plt.close()

# ----- STEP 4b: Save as Table -----
fig, ax = plt.subplots(figsize=(6, 4))
ax.set_axis_off()
table = plt.table(cellText=allocation.astype(int),
                 rowLabels=["S1", "S2", "S3", "S4"],
                 colLabels=["D1", "D2", "D3", "D4"],
                 loc='center',
                 cellLoc='center')

plt.title(f"Optimal Allocation Table\nMinimum Total Cost: {int(total_cost)}", fontsize=12)
plt.savefig("optimal_allocation_table.png", dpi=300)
plt.close()

else:
    print("Optimization failed:", result.message)

```

Optimal Allocation Matrix:

```

[[ 0. 10. 20.  0.]
 [ 5. 20.  0.  0.]
 [20.  0.  0.  0.]
 [ 5.  0.  0. 10.]]

```

Minimum Total Cost: 410.0

Figure 2. Test results using Python's linprog() function

Error analysis was performed by comparing the MODI and Stepping Stone results with the results from the linprog() function (Linear Programming Solver), and a deviation of less than 0.5% was found, confirming the validity of this method against the global optimization approach.

### 3.2. Discussion

The results of this study indicate that the North West Corner (NWC) method can provide a feasible initial solution to the transportation problem, but tends to produce higher distribution costs because it does not consider cost variables in its allocation. This is in line with the findings of [24] which stated that NWC is only effective as an initial method and generally produces solutions that are far from optimal if not followed by an improvement method. In this study, the initial solution using NWC resulted in a total cost of 540, which can then be significantly optimized using the MODI and Stepping Stone methods.

The Modified Distribution (MODI) method has proven to be very effective in evaluating and improving initial solutions through a mathematical approach based on potential values (dual variables) and the opportunity cost of each empty cell. This finding is in line with the research of [11] which states that MODI is a computationally efficient method in finding optimal solutions compared to other methods such as Least Cost or Vogel's Approximation. In the case of more complex distributions, [25] shows that MODI has advantages in terms of the number of iterations and execution time compared to other methods that require manual evaluation at each step.

Furthermore, the application of the Stepping Stone method in this study also succeeded in significantly reducing distribution costs through a visual approach to closed loops. Although this method requires more complex tracking, the results are close to—and in many cases identical to—the results of MODI. This supports the findings of [26] which state that Stepping Stone can achieve the same optimal results as MODI, but with higher complexity in practical application. In this study, the combination of MODI and Stepping Stone was able to reduce the total cost from 540 to 410, reflecting a cost efficiency of 24%.

Different from previous studies that generally only compare two methods partially, such as NWC with Least Cost [18] or VAM with MODI [22], this study provides a comprehensive evaluation of all

three methods simultaneously on one distribution case. This strengthens the contribution of this study in filling the literature gap identified by [15], namely the lack of studies that integrate cost effectiveness, iteration efficiency, and calculation complexity in one analytical framework.

Thus, the findings of this study not only confirm the superiority of MODI and Stepping Stone methods in generating optimal solutions, but also emphasize the importance of a combination strategy between a fast initial allocation method (such as NWC) and an advanced optimization method to achieve maximum distribution efficiency. The practical implications are highly relevant for logistics managers and operational analysts in selecting the most appropriate transportation problem-solving approach given the characteristics of the problem and the limitations of available resources.

#### 4. CONCLUSION

The results of this study indicate that the North West Corner (NWC) method is only effective as an initial solution due to its ease of allocation, but results in high distribution costs because it does not consider cost elements. The application of the Modified Distribution (MODI) method has been proven to systematically optimize the initial solution through the evaluation of potential value and opportunity costs, while the Stepping Stone method provides additional refinement through the visual evaluation of closed paths. The combination of MODI and Stepping Stone effectively reduces total distribution costs from 540 to 410, achieving over 24% efficiency improvement compared to the initial solution. In simple cases (limited number of sources and destinations), MODI demonstrates advantages in convergence speed and ease of implementation, while Stepping Stone is more suitable for complex cases due to its ability to thoroughly explore redistribution through closed paths, despite higher computational complexity. This study only uses one scenario with fixed distribution costs and does not consider demand dynamics or real-time cost variations. These conditions may limit the generalization of results to distribution contexts with fluctuating costs or probabilistic stock demand. The practical implication of this study is to provide guidelines for selecting transportation problem-solving methods based on the complexity of the problem. For further research, it is recommended to develop this method in dynamic cost models, real-time logistics scenarios, or cases with probabilistic demand to assess the performance of the method under conditions of uncertain demand and time.

#### REFERENCES

- [1] F. Saldanha-da-Gama and S. Wang, "Logistics and Supply Chain Management," in *Facility Location Under Uncertainty: Models, Algorithms and Applications*, Springer, 2024, pp. 371-413. doi: 10.1007/978-3-031-55927-3\_12.
- [2] M. Vasileiou *et al.*, "Digital Transformation of Food Supply Chain Management Using Blockchain: A Systematic Literature Review Towards Food Safety and Traceability: M. Vasileiou *et al.*," *Bus. Inf. Syst. Eng.*, pp. 1-28, 2025, doi: 10.1007/s12599-025-00948-0.
- [3] Y. Crama, M. Grabisch, and S. Martello, "Still more surveys in operations research...," *Ann. Oper. Res.*, vol. 343, no. 2, pp. 559-571, 2024, doi: 10.1007/s10479-024-06393-8.
- [4] M. M. Ahmed, A. R. Khan, M. S. Uddin, and F. Ahmed, "A new approach to solve transportation problems," *Open J. Optim.*, vol. 5, no. 1, pp. 22-30, 2016, doi: 10.4236/ojop.2016.51003.
- [5] F. A. Wireko *et al.*, "Results in Control and Optimization The maximum range method for finding initial basic feasible solution for transportation problems," *Results Control Optim.*, vol. 19, no. April, p. 100551, 2025, doi: 10.1016/j.rico.2025.100551.
- [6] B. Amaliah, C. Fatichah, and E. Suryani, "A new heuristic method of finding the initial basic feasible solution to solve the transportation problem," *J. King Saud Univ. - Comput. Inf. Sci.*, vol. 34, no. 5, pp. 2298-2307, 2022, doi: 10.1016/j.jksuci.2020.07.007.
- [7] B. Amaliah, C. Fatichah, and E. Suryani, "Total opportunity cost matrix - Minimal total : A new approach to determine initial basic feasible solution of a transportation problem," *Egypt. Informatics J.*, vol. 20, no. 2, pp. 131-141, 2019, doi: 10.1016/j.eij.2019.01.002.
- [8] M. Carter, C. C. Price, and G. Rabadi, *Operations research: a practical introduction*. Chapman and Hall/CRC, 2018. doi: 10.1201/9781315153223.
- [9] A. Roschyntawati *et al.*, "Application of North West Corner, least cost and Vogel's approximation method for optimizing transportation cost of biodiesel between biofuel company and blending terminal," *AIP*

- Conf. Proc.*, 2023, doi: 10.1063/5.0113524.
- [10] S. A. Tabish, "Operations Research BT - Health Care Management: Principles and Practice," S. A. Tabish, Ed. Singapore: Springer Nature Singapore, 2024, pp. 461-470. doi: 10.1007/978-981-97-3879-3\_21.
- [11] K. Nath, D. Rajeev, D. Debi, and P. Acharjya, "Least - looping stepping - stone - based ASM approach for transportation and triangular intuitionistic fuzzy transportation problems," *Complex Intell. Syst.*, vol. 7, no. 6, pp. 2885-2894, 2021, doi: 10.1007/s40747-021-00472-0.
- [12] S. Dhanasekar, S. Hariharan, and P. Sekar, "Fuzzy Hungarian MODI Algorithm to Solve Fully Fuzzy Transportation Problems," *Int. J. Fuzzy Syst.*, vol. 19, no. 5, pp. 1479-1491, 2017, doi: 10.1007/s40815-016-0251-4.
- [13] D. Schüßler, J. Mantilla-Contreras, R. Stadtmann, J. H. Ratsimbazafy, and U. Radespiel, "Identification of crucial stepping stone habitats for biodiversity conservation in northeastern Madagascar using remote sensing and comparative predictive modeling," *Biodivers. Conserv.*, vol. 29, no. 7, pp. 2161-2184, 2020, doi: 10.1007/s10531-020-01965-z.
- [14] A. K. Bairagi *et al.*, "Coexistence Mechanism between eMBB and uRLLC in 5G Wireless Networks," *IEEE Trans. Commun.*, vol. 69, no. 3, pp. 1736-1749, 2021, doi: 10.1109/TCOMM.2020.3040307.
- [15] U. Ekanayake, W. Daundasekera, and Z. A. M. S. Juman, "A Modified Ant Colony Optimization Algorithm for Solving a Transportation A Modified Ant Colony Optimization Algorithm for Solving a Transportation Problem," *J. Adv. Math. Comput. Sci.*, vol. 35, no. November 2021, pp. 83-101, 2020, doi: 10.9734/jamcs/2020/v35i530284.
- [16] R. H. Kankarofi, U. Ayakubu, I. M. Sulaiman, M. Mamat, Sukono, and M. P. A. Saputra, "Fertilizer Transportation Problem Using Vogel Approximation Method," *IOP Conf. Ser. Mater. Sci. Eng.*, vol. 1115, no. 1, p. 012005, 2021, doi: 10.1088/1757-899x/1115/1/012005.
- [17] H. A. Hussein and M. A. K. Shiker, "A Modification to Vogel's Approximation Method to Solve Transportation Problems," *J. Phys. Conf. Ser.*, vol. 1591, no. 1, 2020, doi: 10.1088/1742-6596/1591/1/012029.
- [18] S. K. Sharma and K. Goel, "Analysis of IBFS for Transportation Problem by Using Various Methods," vol. 10, no. 4, pp. 741-746, 2022, doi: 10.13189/ms.2022.100404.
- [19] K. Karagul and Y. Sahin, "Engineering Sciences Original article A novel approximation method to obtain initial basic feasible solution of transportation problem," *J. King Saud Univ. - Eng. Sci.*, vol. 32, no. 3, pp. 211-218, 2020, doi: 10.1016/j.jksues.2019.03.003.
- [20] U. Ekanayake, W. Daundasekera, and Z. A. M. S. Juman, "An Effective Alternative New Approach in Solving Transportation Problems An Effective Alternative New Approach in Solving Transportation Problems," *Am. J. Electr. Comput. Eng.*, vol. 5, no. January, pp. 1-8, 2021, doi: 10.11648/j.ajece.20210501.11.
- [21] N. Kalaivani and E. M. Visalakshidevi, "Measurement : Sensors A generalized novel approach to transportation problem using multi partite-graph method," *Meas. Sensors*, vol. 33, no. December 2023, p. 101060, 2024, doi: 10.1016/j.measen.2024.101060.
- [22] R. Askerbeyli, "Study of Transportation Problem of Iron and Steel Industry in Turkey Based on Linear Programming, Vam and Modi Methods," *Commun. Fac. Sci. Univ. Ankara Ser. A2-A3 Phys. Sci. Eng.*, vol. 62, no. 1, pp. 79-99, 2020, doi: 10.33769/aupse.740416.
- [23] Y. Muhimpundu, L. O. Odongo, and A. O. Kube, "a Modified Exponentiated Inverted Weibull Distribution Using Modi Family," *Commun. Math. Biol. Neurosci.*, vol. 2025, pp. 1-29, 2025, doi: 10.28919/cmbn/9066.
- [24] T. O. Aliu, Y. O. Aderinto, and K. Issa, "Corner Rules Method of Solving Transportation Problem," *Earthline J. Math. Sci.*, vol. 10, no. 2, pp. 305-316, 2022, doi: 10.34198/ejms.10222.305316.
- [25] M. H. Abdelati, A. M. Abd-El-Tawwab, E. E. M. Ellimony, and M. Rabie, "Solving a multi-objective solid transportation problem: a comparative study of alternative methods for decision-making," *J. Eng. Appl. Sci.*, vol. 70, no. 1, pp. 1-16, 2023, doi: 10.1186/s44147-023-00247-z.
- [26] C. Aroniadi and G. N. Beligiannis, "Solving the Fuzzy Transportation Problem by a Novel Particle Swarm Optimization Approach," *Appl. Sci.*, vol. 14, no. 13, p. 5885, 2024, doi: 10.3390/app14135885.